accordance with Fig. 5, where all variables are now binary. Applying usual Boolean notation, machines M_3 and M_4 may be specified as follows:

$$M_3: t_3 = s_3 \bigtriangleup x_3 = \bar{s}_3 \cdot x_3$$

$$z_3 = s_3 \bigtriangleup x_3 = s_3 \cdot x_3$$

$$M_4: t_4 = s_4 \bigtriangleup (z_3, w) = w \oplus s_1 \cdot \bar{z}_3$$

$$z = s_4 \bigtriangleup (z_3, w) = s_4 \cdot w \cdot \bar{z}_2.$$

The cascade-parallel machine network thus obtained is a widely isomorphic realization of the originally specified machine M (Table II).

CONCLUSIONS

This communication deals with some aspects of sequential machine decompositions. Considerable additional efforts are required to establish a synthesis theory of machine networks which should include more generally applicable results and more efficient procedures for simplifying the component machines.

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Constant Current Source for Analog Computer Use*

I have read with interest the communication by R. W. Thorpe.1

It may interest your readers to know that the circuit described therein first appeared in the "Lightning Empiricist"² in October, 1958. The earlier circuit is reproduced in Fig. 1, together with a circuit employing a differential operational amplifier for a similar purpose.

In Fig. 2, we show yet another circuit capable of supplying an arbitrary current to an arbitrary grounded load-this time employing a differential operational amplifier. This circuit was invented at Lincoln Laboratory, Lexington, Mass., and was first published several years ago.3

The writer has collected a number of voltage-to-current transducer circuits employing operational amplifiers and will be happy to furnish them to interested persons, together with design equations for practical circuits.

It should be noted that the above circuits are capable of stably driving open-circuit stable negative resistive loads (in the sense of Karplus)⁴ if sufficient positive resistance is added in series to ensure that the worst case total load resistance is equal to or greater than zero. If it is desired to read the voltage across the negative resistor, a simple follower

1958.
^a G. A. Philbrick, "Electronic Analog Instruments as Tools of Research and Development," *Research and Development Magazine*, vol. 12, p. 5: October, 1961.
⁴ W. J. Karplus, "Analog Simulation," McGraw-Hill Book Company, Inc., New York, N. Y.; 1958.

e'(1+ -) $+e'(1+\frac{r}{R_1})$ æ (1+=) $\mathcal{E} - \mathcal{E}\left(1 + \frac{\mathbf{r}}{R_{1}}\right) = E_{j} \frac{R}{R_{j}}$ ź. DIFFERENTIAL

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Fig. 1-Driving current to a grounded load.



Fig. 2--Differential operational amplifier as current injector.

circuit may be used, employing a differential operational amplifier.

The controlled current can often be checked by returning the load to an amplifier summing point (virtual ground) and reading the output voltage under resistive feedback, as the load impedance is varied.

Two final thoughts, in the form of semantic quibbles: The terms "arbitrary current generator" or "voltage-to-current transducer" appear to the writer more descriptive of the actual stituation than "constant curent generator." Also, it is hard to visualize a "compliance" having the dimensions of voltage; "maximum rated output voltage" is perfectly acceptable and crystal clear. Perhaps the author was thinking of compliance as a quality of current regulators in quite reasonable analogy to stiffness as a quality of voltage regulators.

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Analog Computer Simulation of a Curved Hysteresis Loop*

In a study of power transformers we have had occasion to simulate the hysteresis characteristic shown in Fig. 1, on an analog computer. Our method may be of interest to others faced with a similar problem.

One half of the loop, $f(v_1)$, is simulated by means of a function generator, giving

$$v_2 = f(v_1)$$
 $(\hat{v}_1 > 0).$ (1)

where

$$\dot{v}_1 = \frac{dv_1}{dt} \cdot$$

* Received February 12, 1963.



Fig. 1-Actual hysteresis loop.



Fig. 2-Hysteresis loop simulator.

Use is then made of the fact that

$$v_2 = g(v_1) = -f(-v_1)$$
 $(v_1 < 0).$ (2)

A simple comparator circuit is used to detect the sign of v_1 and initiate the switching indicated by (2). The set-up is shown in Fig. 2. The circuit has limitations; for example:

- 1) the input v_1 must vary between the exact limits $-v_{1m}$ and $+v_{1m}$, and \dot{v}_1 must change sign only at these values of v_1 ; and
- 2) the input should not have harmonic content above the break frequency of the differentiator, and the speed of operation of the comparator relay is also significant in this regard.

The first limitation is, of course, due to the physical nature of hysteresis in iron, and both limitations make the circuit suitable only for sinusoidal or near-sinusoidal inputs.

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Measurement of Phase Shift*

In analog computer circuitry, the need for accurate phase measurements often arises. Inverting amplifiers contribute significant phase shifts at frequencies well below the 3-db frequency. These phase shifts cause large errors in a problem solution. Conventional phase measuring equipment does not operate at the higher frequencies where high-speed computers now operate. A simple procedure has been developed which utilizes a dual beam oscilloscope, or any indicating device with a

^{*} Received February 19, 1963. ¹ R. W. Thorpe, "Constant Current Source for Analog Computer Use," *IEEE Trans. on Electronic Computers*, vol. EC-11, p. 793; December, 1962. ² "The Lightning Empiricist," G. A. Philbrick Researches, Inc., Boston, Mass., No. 6, p. 3; October, 1958 1958